MASS TRANSFER IN SINGLE PHASE FLOW THROUGH FIXED AND FLUIDISED BEDS

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Received May 11th, 1979

The analysis of the mass transfer mechanism in single phase flow through fixed and fluidised beds is based on the formely published theoretical model and experimental data from literature. The data in the range $5 < Re < 15000$, $0{\cdot}6 < Sc < 13000$ and $0{\cdot}35 \leqq e \leqq 0{\cdot}94$ have been used. The resulting relations are presented.

The mass transfer in single phase flow through fixed and fluidised beds of packing has been studied by numerous authors. An exhausting survey of these studies can be found in the publication by Wakao and Funazkri[1], Surendra and coworkers[2] and Karabelas and coworkers[3]. The majority of relations presented in literature are of empirical nature and they differ both by their form and definition of some basic parameters.

There exist a number of objective problems which prevent the correlation of experimental data in mentioned systems on basis of a deeper analysis of mass transfer mechanism. Regardless the difficulties in determination of geometric surface area of packing, the value of mass transfer coefficient based on this area is a quantity which represents the mean value of a transport process intensity on the surface of particles in the bed. The local values on the surface of an individual particle can reach very different values especially for more complicated geometric shapes such as are Raschig rings, Berl saddles *etc.*

The main problems encountered from the theoretical point of view represent determination of the characteristic linear dimension and specification of hydrodynamic conditions at the fluid flow through the bed. These are the main reasons why the correlation of experimental data is still essentially the empirical problem.

In the following paper the corresponding experimental data are correlated on basis of the earlier published[4] theoretical model, verified in a wide range of variables on the well defined system – a pipe of circular cross section. Though the proposed model does not overcome in the given case the mentioned difficulties it enables verification how suitably some parameters were selected from the point of view of uniform description of transport processes in systems with differing geometry.

THEORETICAL

Theoretical model which is discussed respects the effect of all three familiar hydrodynamic regions on transport processes across the phase interface which are the laminar layer, transition region and region of developed turbulence. The mass or heat transfer coefficients are defined for the quasistationary state, at which a steady transport across the laminar layer and the unsteady transport across the remaining two regions for the time interval given by the time scales of turbulence in the corresponding regions take place.

For the mass transfer coefficient the relation results

$$(k\,d_e/D) = (u\,d_e/\gamma)\,(f/2)^{1/2}\,F(\gamma/D)/[\delta_l^+ \,.\, F(\gamma/D) + \\ + \lambda_p^+ + (1/\sqrt{2})\,\phi(\gamma/D)\,(u d_e/\gamma)^{1/4}\,(f/2)^{1/4}]\,, \qquad (1)$$

where

$$\delta_l^+ = (u^*\,\delta_l/\gamma)\,, \quad \lambda_p^+ = (u^*\,\lambda_p/\gamma)\,, \quad f/2 = \Delta p d_e/(4h\varrho u^2)\,, \qquad (2,a,b,c,)$$

and the functions $F(\gamma/D)$ and $\phi(\gamma/D)$ are derived in the original paper[4] in the form

$$F(\gamma/D) = \\ = 1 + (1/3)\,(\gamma/D) - (2/\pi^2)\,(\gamma/D)\sum_{n=1}^{\infty}(1/n^2)\exp\left(-n^2\pi^2/(\gamma/D)\right) \qquad (3)$$

which converges better for small values of (γ/D) and in the form

$$F(\gamma/D) = (2\sqrt{\pi})\,(\gamma/D)^{1/2}[1 + 2\sqrt{\pi}\sum_{n=1}^{\infty}\mathrm{ierfc}\,(n(\gamma/D)^{1/2})] \qquad (4)$$

which converges better for large values of (γ/D) and

$$\phi(\gamma/D) = 2\mathrm{F}(\gamma/D)/[(\gamma/D)\,.\,f(\gamma/D)] - 1\,, \qquad (5)$$

where

$$f(\gamma/D) = 1 - (8/\pi^2)\sum_{n=0}^{\infty}(1/(2n+1)^2)\exp\left[-(2n+1)^2\,\pi^2/(\gamma/D)\right]. \qquad (6)$$

For the values $(\gamma/D) \leqq 1$ the third right hand side term in Eq. (3) can be neglected. For values $(\gamma/D) \geqq 3$ the second right hand side term in the brackets in Eq. (4) can be neglected and in the region $1 < (\gamma/D) < 3$ the simplified relation can be used

$$F(\gamma/D) = 1{\cdot}32(\gamma/D)^{0{\cdot}358}\,. \qquad (7)$$

For values $(\gamma/D) \leqq 1$ it is possible in Eq. (5) for $\phi(\gamma/D)$ to use the relation $f(\gamma/D) = 1$ and for $(\gamma/D) \geqq 10$ to use the relation $\phi(\gamma/D) = 0$. In the case of mass transfer in the bed of solid particles such conditions might occur that the mentioned quasistationary state is not established *i.e.* the corresponding velocity and concentration profiles are not developed on the interface during the flow around the element of the bed. In such a case it is possible to expect that the conditions for mass transfer will fit better the model according to Higbie of unsteady transfer for which the relation holds

$$k \approx (D/t_e)^{1/2}, \tag{8}$$

where for t_e it is possible in the first approximation to substitute

$$t_e = (d_p/u^*), \tag{9}$$

so that after arrangement into the dimensionless form the relation is obtained

$$(kd_e/D) \approx [(d_e/d_p)(u^* d_e/\gamma)(\gamma/D)]^{1/2}. \tag{10}$$

Which of these relations is considered in any particular case is given both by Reynolds and Schmidt criteria and by the linear dimension of the element of the bed.

RESULTS AND DISCUSSIONS

For verification of the given model assumptions the experimental data published in literature were used. An attempt has been made to obtain such systems of data where beside the own mass transfer data are also included data on pressure drop which are needed for calculation of the friction velocity or of the friction factor. As such systems of data are relatively scarce such sets of systems were used for which the pressure drop could have been calculated from the familiar relations.

As the linear characteristic dimension was used the usual definition of the equivalent dimension of channels in the bed in the form

$$d_e = d_p e/(1 - e) \tag{11}$$

and as the characteristic velocity the velocity in channels in the form

$$u = (u_b/e). \tag{12}$$

In the first approximation the same values for the dimensionless thicknesses of the laminar, and transition regions $\delta_l^+ = 1$ and $\lambda_p^+ = 20$ were used as determined in the last study[4] where the pipe of circular cross section, has been considered. It was

assumed that their possible modification will result during the performance of correlation.

In Table I is given the survey of evaluated experimental data. From this Table it is obvious that the data include the dissolving, evaporation and sublimation processes, while dissolving has been performed both in the fixed and fluidised beds. The whole range of Reynolds numbers is $5 < \mathrm{Re} < 15000$ and of the Schmidt number is $0{\cdot}6 < \mathrm{Sc} < 13000$. Porosity of the bed has been altered in the range $0{\cdot}35 < e < 0{\cdot}778$ with the fixed bed and up to 0·94 for the fluidised bed.

In evaluation of the experimental data the approach has been used in which all data were at first evaluated according to Eq. (*1*) and on basis of the plot of data in the form of $\mathrm{Sh}\,[1 + 20/F(\gamma/D) + 0{\cdot}707(\phi(\gamma/D)/F(\gamma/D))\,(\mathrm{Re}\,f/2)^{1/4}]$ on

$$\mathrm{Re}\,(f/2)^{1/2}$$

such data were eliminated which with a sufficient accuracy did not fit the relation (*1*) and such data were evaluated according to Eq. (*10*). The both mentioned dependences

TABLE I

Summary of Evaluated Experimental Data

Authors	Experimental conditions		Experimental system		
	operation	bed	solid phase		fluid phase
			material	shape	
McCune and Wilhelm[5]	dissolving	fixed fluidised	naphthalene	spherical pellets	water
Gaffney and Drew[6]	dissolving	fixed	salicyl and succinic acids	spherical pellets	benzene n-butyl-alcohol, acetone
McConachie and Thodos[7]	evaporation of water	fixed	porous	spherical	air
Petrovic and Thodos[8]	evaporation n-decane n-octane n-dodecane n-tetradecane	fixed	porous	spherical	air
Shulman and coworkers[9]	sublimation	fixed	naphthalene	Raschig rings Berl saddles	air

for the bed consisting of particles of more or less spherical shape are plotted in Fig. 1. The dependence according to Eq. (*1*) for Raschig rings and Berl saddles is plotted in Fig. 2.

By statistical evaluation of 157 data of the first set of data which are plotted in Fig. 1 and 69 data of the second set plotted in Fig. 1 the following relations were obtained

$$\mathrm{Sh} = 1{\cdot}636[\mathrm{Re}(f/2)^{1/2}]^{0{\cdot}933}/[1 + 20/F(\gamma/D) + \\ + 0{\cdot}707(\phi(\gamma/D)/F(\gamma/D)) \,.\, (\mathrm{Re}(f/2))^{1/4}] \qquad (13)$$

with the mean relative deviation 17·02% for the first set and in the form

$$\mathrm{Sh} = 0{\cdot}317[\mathrm{Re}(f/2)^{1/2}]^{0{\cdot}512} \,.\, (\mathrm{Sc}\, e/(1 - e))^{1/2} \qquad (14)$$

with the mean relative deviation 17·78% for the second set.

TABLE I
(*Continued*)

Dimensions in mm		Range of experimental parameters		
bed	d_p	Re	Sc	e
∅ 101·6	6·35; 4·76; 3·175	20—2 800	1 200— 1 500	0·35 —0·94
∅ 76·2	6·34; 6·4; 9·51; 12·44	0·9—1 555	150—13 000	0·37 —0·62
∅ 93·78	15·875	245—7 285	0·607	0·416—0·778
177·8 × 177·8	1·83; 2·18; 2·6; 3·09; 9·4	5—438	0·6—5·45	0·394—0·475
∅ 254	17·6; 53·04; 16·21; 32·00	200—15 000	2·52	0·630—0·695

By evaluation of the first set of data (denoted by *1* in Fig. 1) at the assumption that the exponent over $\mathrm{Re}\,(f/2)^{1/2}$ is equal to one *i.e.* the theoretical value, the constant 1·636 in Eq. (*13*) will change to the value 1·237 and the mean relative deviation to 17·45%. Similarly by evaluation of data in the second set (denoted by 2 in Fig. 1) at the assumption that the exponent over $\mathrm{Re}\,(f/2)^{1/2}$ is equal to 0·5 as results from a theoretical consideration the constant in the relation (*14*) will change from 0·317 to 0·353 and the mean relative deviation to 17·37%. Thus the relations (*13*) and (*14*) obtained by evaluation of experimental data with a sufficient accuracy are in agreement with the theoretically predicted dependences (*1*) and (*10*).

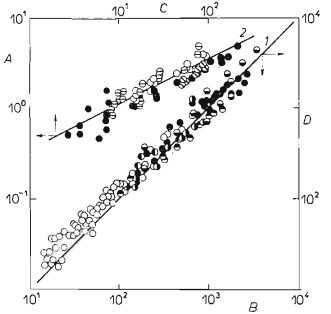

FIG. 1

Plots of Experimental Correlations of Experimental Data for Spherical Particles

Correlation according to Eq. (*1*): $B = \mathrm{Re}\,(f/2)^{1/2}$, $D = \mathrm{Sh}\,[20 + F(\gamma/D) + 0{\cdot}707\phi(\gamma/D)\,(\mathrm{Re}\,f/2)^{1/4}]/F(\gamma/D)$, ◓ fixed bed[5], ● fluidised bed[5], ◐ benzene salicylic acid[6], ◑ acetone/succinic acid[6], ◒ fixed and distended bed[7], ○[8], *1* theoretical relation. Correlation according to Eq. (*10*), $A = \mathrm{Sh}/[\mathrm{Sc}\,.\,e/(1-e)]$, $C = \mathrm{Re}(f/2)^{1/2}$. The point with way line naphthalene/water[5], acet one/succinic acid[6], ● butylalcohol/succinic acid[6], ⊖ benzene/salicylic acid[6], *2* theoretical relation with the exponent 0·5 over $\mathrm{Re}\,(f/2)^{1/2}$.

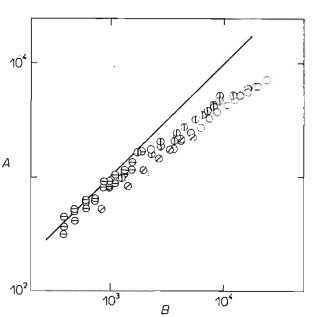

FIG. 2

Plot of Correlations of Experimental Data for Raschig Rings and Berl Saddles According to Eq. (*1*)

$B = \mathrm{Re}(f/2)^{1/2}$, $A = \mathrm{Sh}[20 + F(\gamma/D) + 0{\cdot}707\phi(\gamma/D)\,(\mathrm{Re}\,f/2)^{1/4}]/F(\gamma/D)$. Symbols used are identical with numbering of references. ⊙ Berl saddles 1″ (ref.[9]), ⊖ Berl saddles 1/2″ (ref.[9]), ○ Raschig rings 1″ (ref.[9]). The point with way line Raschig rings 1/2″ (ref.[9]). Solid line represents the theoretical relation.

From relations (*13*) and (*14*) by use of theoretical exponents over $\mathrm{Re}\,(f/2)^{1/2}$ and by use of corresponding constants it is possible to derive the relation for the critical value of $\mathrm{Re}\,(f/2)^{1/2}$ which delimits the validity of individual relations in the form

$$[\mathrm{Re}\,(f/2)^{1/2}]_{\mathrm{Kr}} = 0{\cdot}08(\mathrm{Sc}\,e/(1-e))\,[1 + 20/F(\gamma/D) + \\ + 0{\cdot}707(\phi(\gamma/D)/F(\gamma/D))\,(\mathrm{Re}\,f/2)_{\mathrm{Kr}}^{1/4}]^2\,, \qquad (15)$$

where for $(\gamma/D) \geqq 10$ it is possible to neglect the third right hand side term in brackets. For the value $\mathrm{Re}\,(f/2)^{1/2}$ larger than the critical one relation (*13*) holds, otherwise the relation (*14*). The lower limit of validity of relation (*14*) and the upper limit of validity of relation (*13*) are not known.

The third set of data which is plotted in Fig. 2 demonstrates that the data are in agreement with the theoretical dependence only up to some value $\mathrm{Re}\,(f/2)^{1/2}$ and then they significantly deviate. This deviation can be explained by complex form of elements of packings used (Raschig rings and Berl saddles) so that the mass transfer coefficients calculated by use of geometrical area of the bed do not respect the fact that the local mass transfer coefficients over the geometrical surface of elements of the bed can mutually differ significantly and might vary with the value of $\mathrm{Re}\,(f/2)^{1/2}$.

CONCLUSIONS

By evaluation of a large set of data in a wide range of operating variables the obtained correlation relations (*13*) and (*14*) fit with a sufficient accuracy the theoretical relations (*1*) and (*10*). It is especially interesting that the relation (*1*) with the exception of a relatively small change in the constant included holds for so different systems as are the bed of particulate material and the pipe of circular cross section. Variation in the constant from 1 to 1·237 is perhaps due to difference in assumed values of δ_1^+ and λ_p^+ which could differ from their actual values, but their experimental determination from experimental data would not have any sense as concerns the correlation.

LIST OF SYMBOLS

d_e equivalent dimension of channels of the bed, defined by Eq. (*11*) (L)
d_p equivalent dimension of the element of the bed (diameter of a sphere having the same surface area as the element of the bed) (L)
D diffusivity (L^2/T)
e porosity
f friction factor, def. by Eq. (*2c*)
$f(\gamma/D)$ function defined by Eq. (*6*)
$F(\gamma/D)$ function defined by Eqs (*3*) and (*4*)

h height of bed (L)
$\text{ierfc}\, x = \int_x^\infty \text{erfc}\, \xi \, d\xi = (1/\sqrt{\pi}) \exp(-x^2) - x \,\text{erfc}\, x$
k mass transfer coefficient (L/T)
Δp pressure drop (M/LT^2)
u velocity in channels, def. by Eq. (*12*) (L/T)
u_b superficial velocity (L/T)
$u^* = u\sqrt{(f/2)}$ friction velocity (L/T)
t_e time of contact (T)
δ_l thickness of laminar layer (L)
δ_l^+ dimensionless thickness of laminar layer, def. by Eq. (*2a*)
λ_p thickness of transition region (L)
λ_p^+ dimensionless thickness of transition region, def. by Eq. (*2b*)
γ kinematic viscosity (L^2/T)
$\phi(\gamma/D)$ function defined by Eq. (*5*)
ϱ specific density (M/L^3)
$\text{Re} = ud_e/\gamma = u_b d_p/(1-e)\,\gamma$ Reynolds number
$\text{Sc} = \gamma/D$ Schmidt number
$\text{Sh} = k \,.\, d_e/D = k \,.\, d_p e/D(1-e)$ Sherwood number

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Translated by M. Rylek.